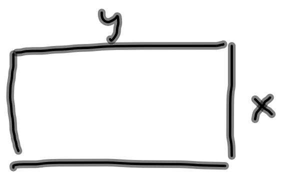
1.6 Mathematical Models: Constructing Functions

7. **Enclosing a Rectangular Field** David has available 400 yards of fencing and wishes to enclose a rectangular area.
   (a) Express the area A of the rectangle as a function of the width x of the rectangle.
   (b) What is the domain of A?
   (c) Graph $A = A(x)$ using a graphing utility. For what value of x is the area largest?



Have 400 ft^2 of fencing.

$2x + 2y = 400$

$x + y = 200$

$y = 200 - x$

$yx = \text{area}$

$(200-x)x = A(x)$

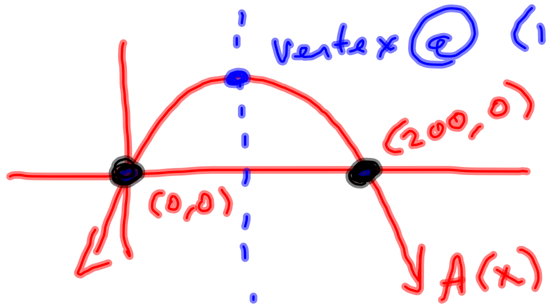
$\mathcal{D}(A)$ would be $\mathbb{R}$ if we weren't limited in the fence available.

$\boxed{A(x) = x(200-x)}$

$\mathcal{D}(A) = \{x \mid 0 < x < 200\}$

$= 200x - x^2 = -x^2 + 200x$

vertex @ $(100, A(100))$



$A(100) = 100(200-100)$

$= 100^2 \text{ ft}^2$

$= 10000 \text{ ft}^2$

10. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$.

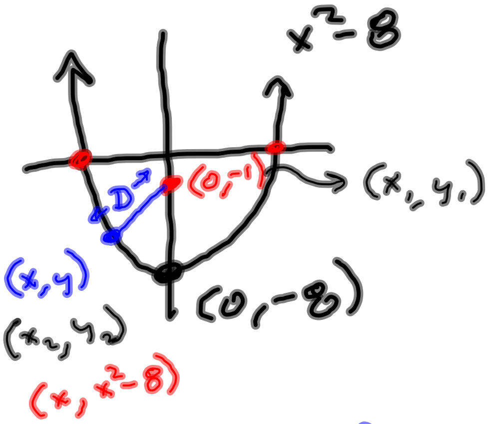
(a) Express the distance d from P to the point $(0, -1)$ as a function of x.

(b) What is d if $x = 0$? $D = 7$

(c) What is d if $x = -1$?

(d) Use a graphing utility to graph $d = d(x)$.

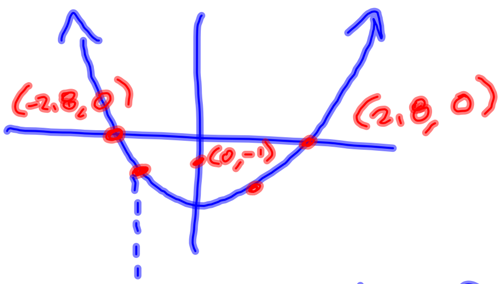
(e) For what values of x is d smallest?



$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{(0 - x_2)^2 + (-1 - y_2)^2}$$
$$= \sqrt{(0 - x)^2 + (-1 - (x^2 - 8))^2}$$
$$= \sqrt{(-x)^2 + (-1 - x^2 + 8)^2}$$
$$= \sqrt{x^2 + (-x^2 + 7)^2}$$
$$= \sqrt{x^2 + (x^4 - 14x^2 + 49)}$$
$$= \sqrt{x^4 - 13x^2 + 49}$$

$$(-x^2 + 7)^2 = (-1(x^2 - 7))^2 = (-1)^2(x^2-7)^2 =$$
$$= (x^2 - 7)^2$$
$$= x^4 - 14x^2 + 49$$

$$(ab)^c = a^c b^c$$




```
                99997
Ans/(100000-1)
          .9999799998
√(5
          2.236067977
√(8)
          2.828427125
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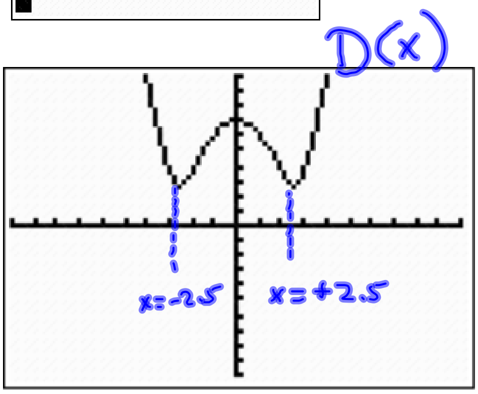
$x \approx -2.5$ is where D seems to be minimized, from graph of $D(x)$



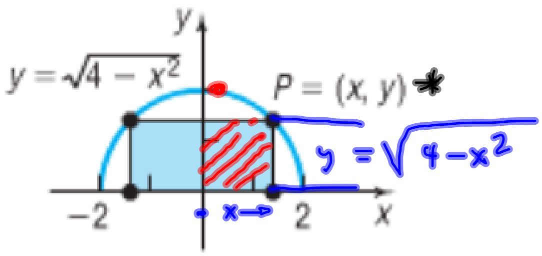
$D(-1) \approx 6.0828$

Using TABLE with TI-84

X	Y2	
-1	6.0828	
1	6.0828	

X=

16. A rectangle is inscribed in a semicircle of radius 2 (see the figure). Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.



$x^2 + y^2 = 2^2$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

$y = \sqrt{4-x^2}$ is top half.

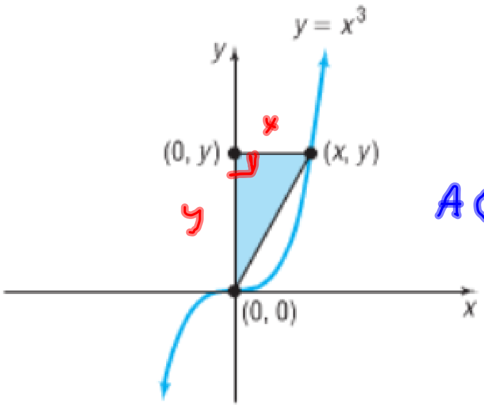
(a) Express the area A of the rectangle as a function of x.
(b) Express the perimeter p of the rectangle as a function of x.
(c) Graph $A = A(x)$. For what value of x is A largest?
(d) Graph $p = p(x)$. For what value of x is p largest?

(a) $A = 2xy = 2x\sqrt{4-x^2} = A(x)$

(b) $P = 2x + 2y = 2x + 2\sqrt{4-x^2} = P(x)$

Graph is homework thing, not a test thing.

13. A right triangle has one vertex on the graph of $y = x^3$, $x > 0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$, as shown in the figure. Express the area A of the triangle as a function of x.

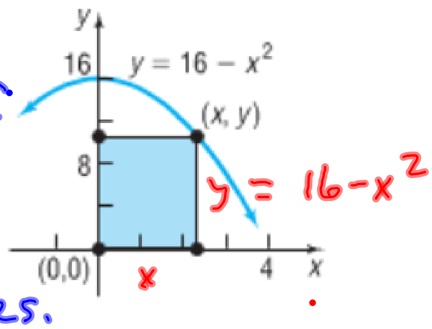


$$\text{Area} = \frac{1}{2} x\, y$$
$$= \frac{1}{2} x (x^3)$$
$$A(x) = \frac{1}{2} x^4$$

15. A rectangle has one corner on the graph of $y = 16 - x^2$, another at the origin, $\boxed{\text{a third on the positive } y\text{-axis,}}$ and the fourth on the positive x-axis (see the figure).

$\mathcal{D} = [0,4]$

OR $(0,4)$ if you exclude degenerate rectangles.



Area $= xy$

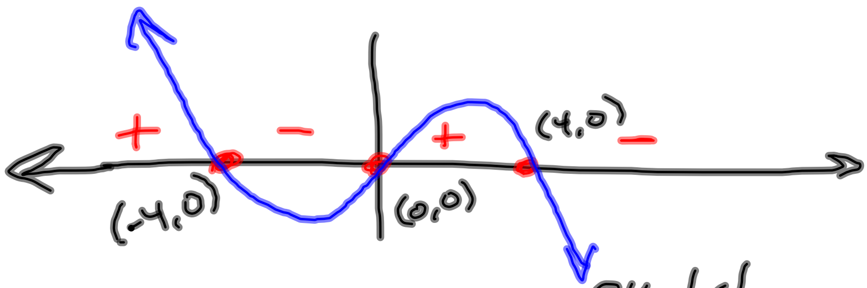
$= x(16-x^2)$

$= A(x)$

(a) Express the area A of the rectangle as a function of x.
(b) What is the domain of A?
(c) Graph $A = A(x)$. For what value of x is A largest?

Sketch of $A(x)$, no calculator:

$x(16-x^2) = x'(4-x)'(4+x)'$



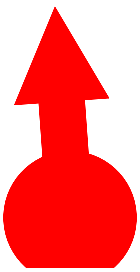
Sketch of $x(16-x^2)$

check sign of $A(5) =$

$5(4-5)(4+5)$

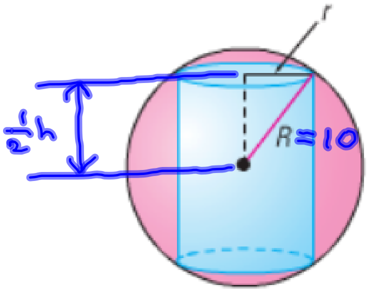
$= 5(-1)(9)$

$= -$

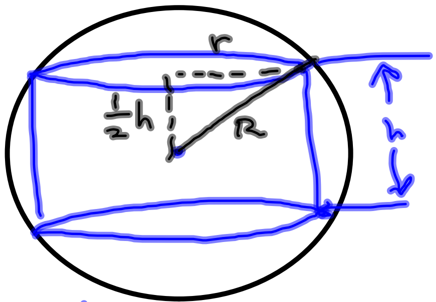


28. Inscribing a Cylinder in a Sphere Inscribe a right circular cylinder of height h and radius r in a sphere of fixed radius R. See the illustration. Express the volume V of the cylinder as a function of h.
[Hint: $V = \pi r^2 h$. Note also the right triangle.]

R is fixed,
r is variable.



Sphere



$V = \pi r^2 h$ Need to ditch the r, somehow.

Pythagorus says

$$\left(\tfrac{1}{2}h\right)^2 + r^2 = R^2$$

$\left(\tfrac{1}{2}\right)^2 (h)^2$

$$r^2 = R^2 - \tfrac{1}{4}h^2$$

$$r = \pm\sqrt{R^2 - \tfrac{1}{4}h^2}$$

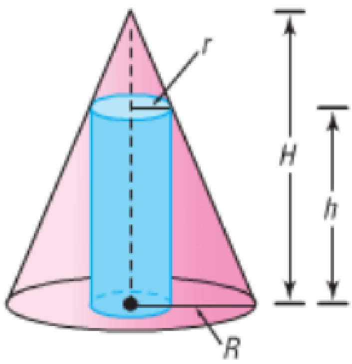
$$r = \sqrt{R^2 - \tfrac{1}{4}h^2}$$

Hence, $V = \pi\left(\sqrt{R^2 - \tfrac{1}{4}h^2}\right)(h)$

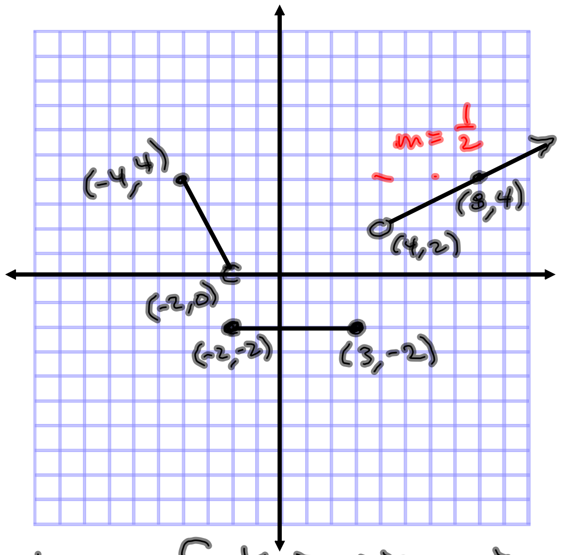
$$= \pi h \sqrt{R^2 - \tfrac{1}{4}h^2}$$

29. Inscribing a Cylinder in a Cone Inscribe a right circular cylinder of height h and radius r in a cone of fixed radius R and fixed height H. See the illustration. Express the volume V of the cylinder as a function of r.

[Hint: $V = \pi r^2 h$. Note also the similar triangles.]



Cone



Definition of this piecewise-defined function.

$$f(x)=\begin{cases} -2x-4 & -4 \le x < 2 \quad (1) \\ -2 & -2 \le x \le 3 \quad (2) \\ \frac{1}{2}x & 4 < x \quad (3) \end{cases}$$

(1) $(-4,4)=(x_1, y_1)$
$(-2,0) = (x_2, y_2)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-4}{-2-(-4)} = \frac{-4}{-2+4} = -\frac{4}{2} = -2 = m$$

(i) $y = mx+b$
$y=-2x+b$
$0=-2(-2)+b$
$0=4+b$
$-4=b$

(2) No work

(ii) $y = m(x-x_1)+y_1$
$y=-2(x--4)+4$
$=-2x-8+4$
$=-2x-4$

(3) $m=\frac{1}{2}$ by inspection
$y=mx+b$
$y=\frac{1}{2}x+b$
$2=\frac{1}{2}(4)+b$
$2=2+b$
$0=b$